The Mathematics of Risk

With insurers adopting a more technical approach to underwriting, the old rules of what constitutes a 'good risk' are changing. John Birkenhead, an independent consulting actuary, answers some common questions from AIRMIC members.

"I've had no claims for a long time - why has my premium gone up?"

"I thought that premium equals cost of risk; I'm a good risk so why does my premium keep going up?"

"I've had no claims this year - can I have my premium back?"

"Why do insurers quote such a wide range of premiums for the same risk? Surely all insurers should quote roughly the same price for the same risk?"

We will answer the above questions by sneaking a quick look "under the mathematical bonnet" at some of the mathematical aspects of setting premiums. In doing so, we will see that saying that "I'm a good risk" does not address insurers' concerns, and so we will also briefly look at ways to improve insureds' presentation of their risks.

The Classic Car Insurance Problem

We will introduce the mathematical concepts with a typical car insurance problem as shown below:

How much premium would you charge for the following risk?

- 18 year-old male driver
- Drives X-Reg Ford Fiesta, valued at £1,000
- Has 3 penalty points on licence for speeding
- Lives in central London
- Wants third party cover only, for 12 months

The typical insurance problem may be summarised as follows:

- Each insured pays a premium (usually different for each insured) to the insurer
- The premium is charged in advance and is fixed at the outset of the policy
- The insurer will then pay for all damage caused by the insured to third parties (and their property) during the policy period (usually the following 12 months)

The car insurance problem is typically solved as a so-called "multivariate" problem; that is to estimate the risk premium (equals claim frequency times claim severity) for each combination of factors collected at the point of sale (such as age of driver, value of car etc). The actuarial mathematics behind this solution is highly complex, and we do not need to go into it here, but with millions of policies and claims, insurers can quote "individual" premiums based on the unique characteristics of each risk.

Corporate Insurance - Just do the maths!???

Surely then, for corporate insurance, it is just a questions of "doing the maths"? Not quite. Corporate insurance is much more complex for the reasons shown below:

- Policy excesses are typically £50,000-£50,000,000 per claim (or more) – many claims are not reported to the insurer at all; even those which are reported may not even reach the excess
- The maximum claim from a single insured probably is c£10 billion or more (e.g. oil pollution, catastrophic explosion, earthquake, flood etc)– this level of cover has severe capital requirements for the insurer (see below)
- The insured risks are usually very diverse risks in different countries, different currencies, constant acquisitions, disposals etc – each risk cannot be assessed in detail – the insurer's overall aim is the adequacy of the premium for the insured (policy) as a whole

We will now look in more detail at some of the issues for policies bought by AIRMIC members.

Property Catastrophe (CAT) Cover

Property CAT typically covers property claims in the extreme tail of the underlying distribution, such as claims excess of a (real-life) deductible of £50million. Such claims are clearly extremely rare and so "intense mathematical" approaches (such as those above) fail due to the extreme sparsity of claims data. Such rare events are often modelled mathematically using Generalised Extreme Value (GEV) distributions fitted to the extremely limited historic data. GEVs are an evolving area of mathematical research. As new events occur, premiums for such covers inevitably rise as the modeling teams "re-calibrate" their CAT models (e.g. recent UK floods, US hurricanes etc).

Employers' Liability (EL)

The main historic problem for EL covers has been asbestos-related claims, and in particular mesothelioma claims, which have caused the downfall of a number of corporate insurers. These claims have generally arisen out of periods of employment (and hence insurance coverage) in the 1950s and 1960s. Mesothelioma claims usually cost £100,000-£300,000 each. There have been various mathematical/epidemiological models over the years to try to forecast future claims, and these evolve over time to better "fit" the historical data. Although asbestos is not used to the same extent today (at least in the UK), insurers are wary of other similar "dormant" diseases which may be caused by employment conditions; perhaps work-related stress, repetitive strain injury for younger workers, wireless offices etc. will be the new "asbestos" problem for insurers in 40 years' time?

Put another way, most insureds will be expected to have no such claims for perhaps 10-20 years after the insurance period – and the insurer generally cannot go back to the insured to ask for additional premium.

So what for insurers?

The Financial Services Authority requires insurers to hold sufficient capital to remain solvent over a 12-month period at the "99.5th percentile" (a "1 in 200 year" event) of the joint aggregate distribution (expected "behaviour") of its assets and liabilities.

In other words, if the insurer writes business which is very uncertain (e.g. Property CAT, EL), the regulator will require a higher level of capital support; in addition to which the capital providers will inevitably require a higher return on capital to compensate for the realistic risk of capital loss.

The Real Premium Calculation – maths for the terrified!

In simple terms (ignoring profit, solvency etc), an insurer's total premium (across all classes) is designed to equal its total outgo (across all classes). If an insurer has 10 insureds (each paying a premium of 10), and expects claims and of 100 each year from 1 out of the 10 insureds, then there will be sufficient premiums to pay all the claims. If each non-claiming insured gets their premium back, everyone would need to be charged 100 (instead of 10) so that total premiums (1,000) = total claims (100) + total refunds (900).

For corporate insurance a premium refund is even less likely; for CAT claims (which are generally 1 in N year events) the total premium collected in any one year would only be 1/Nth of the cost of a CAT claim. At a simple level, CAT insurers rely on collecting premiums for N-1 claim-free years to be able to pay for the 1 in N year event (although the maths, accounting and solvency issues are much more complex than this).

In more complex terms, an insurer's total premium (across all classes) is designed to equal its total outgo (across all classes) plus:

Shareholders' required return on capital, which in turn, is equal to:

- The amount of capital required to satisfy the "1 in 200 year" regulatory solvency threshold (the "99.5th percentile" joint aggregate distribution test)
- The risk-free rate of return shareholders could obtain by investing their money risk-free elsewhere (e.g. medium-term gilts, say 5% p.a.)
- The additional "risk" element of the rate of return required by shareholders (for example, an additional 10-25% p.a. depending upon the class of business); based on the realistic risk of losing some of their capital investment due to, for example, CAT claims.

In other words, the premium charged to an individual insured can therefore rise (or fall, sometimes) for reasons not directly related to the insured's claims experience - for example, increasing frequency/severity of large losses for similar insureds (e.g. the "1 in 200 year" event is actually more frequent than expected), increased capital requirements from regulators, increased return on capital requirements from the insurer's shareholders/capital providers etc.

Insureds are essentially "renting" the insurer's capital in the event of a large loss, (much the same as paying an "overdraft arrangement fee" at your bank, even if you do not ever use the overdraft - the overdraft fee is essentially for the bank's "underwriting" of your risk).

Answering the common questions

"I've had no claims for a long time - why has my premium gone up?"

For corporate insurance, due to the very large claim sizes involved, there is not much direct link ("statistical credibility") given to the claims data from a single insured. Premiums can therefore rise for reasons not directly related to the insured's claims experience as described above. Especially for CAT covers, "having no claims for a long time" is to be expected from most insureds.

"I thought that premium equals cost of risk; I'm a good risk so why does my premium keep going up?"

Premium equals the cost of (one-form of) risk transfer; it does not equal the cost of risk. Premiums can therefore rise for reasons not directly related to the insured's claims experience as above. ▼

"I've had no claims this year - can I have my premium back?"

Insurance is a form of risk-sharing; premiums (from all insureds) are used to pay for the claims from insureds who do claim. Initial premiums would have to rise by several hundred percent to allow full premium refunds.

"Why do insurers quote such a wide range of premiums for the same risk? Surely all insurers should quote roughly the same price for the same risk?"

Premiums can differ substantially for reasons not directly related to the actual risk being presented, but instead reasons related to the regulatory/solvency/return on capital demands of those providing the capital to be "rented" out to insureds. For example, a property CAT insurer with a balanced, diversified book of business will, in general, have a lower capital requirement (and therefore probably lower premiums, depending upon shareholders' required return on their capital) than a monoline insurer focusing only on a specific type and geography of property CAT claims (e.g. UK floods).

So how do you really present a good risk?

Here are some good tips for engaging with insurers (and the mathematicians and actuaries who work "behind the scenes" for them to set premiums):

- Tell them about the risks - and potential large claims - they face next year, not the risks (and claims) they faced last year (which they already know about). They are concerned about volatility of profits, minimizing capital requirements and aggregations of exposures for your potential large claims.
- What are realistic large losses for your organization (e.g. maximum probable losses)? What are you doing to prevent such large losses? What will you do in the event of such a loss to keep the costs down for the insurer?
- Your past claims will be more expensive if they happen next year (claims inflation), but the deductibles may well be the same. So some claims which were below deductibles last year will be above (the same) deductibles this year.
- Give them some metrics to measure claims (e.g. cost per employee) – but allow for acquisitions and disposals; claims may be falling, but not as fast as exposures. If you dispose of a division with a "good" risk profile, the rest of your risk becomes "worse" than it was.

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